



## A STATIC/DYNAMIC PROCEDURE FOR COLLAPSE ANALYSIS OF STRUCTURE IN FIRE

*Ruirui Sun, Zhaohui Huang, Ian Burgess*  
*University of Sheffield, Department of Civil and Structural Engineering*

### ABSTRACT

In this paper a recently developed static/dynamic procedure, which extends the capability of the Vulcan software to progressive collapse analysis, is described. This can also be utilized to allow a basically static analysis to continue beyond temporary instabilities, due for example to major concrete cracking or to failure of an individual bolt in a connection, which would cause singularities in static analyses. The procedure is validated against several practical cases. It will be applied to progressive collapse analysis of steel/composite structures under fire conditions for further research.

### 1. INTRODUCTION

The structural behaviours affected by fire are so complicated that sometimes tests and analytical results based on individual elements are not sufficient, and full-scale whole-frame modelling is really necessary. In such cases, numerical modelling software is becoming an indispensable tool. Vulcan, an in-house developed finite element software, has been validated as being capable of handling the required nonlinear static structural analysis under elevated temperatures, but in order to investigate the robustness of a structure, a dynamic procedure is essential to trace the structural behaviour up to collapse. This study focuses on development of the fully dynamic, and the static/dynamic versions of Vulcan.

### 2. DYNAMIC PROCEDURE

The integration method of time-history analysis can be divided into two groups, implicit methods and explicit methods. In an implicit solution, global equilibrium is first achieved by iteration, following which local elemental variables are evaluated. By contrast, explicit solution evaluates local variables directly without the need for global equilibrium calculation. Since implicit dynamic procedure requires forming and inverse the global stiffness matrix, hence, more disk space and memory are needed compared to explicit dynamic. Thus, for large scale problem explicit dynamic will be more effective than implicit one. Moreover, for problems with high nonlinearity or material complexity, the implicit dynamic would have difficulty to get a converged solution, resulting in either a large number of iterations needed or numerical failure of the analysis.

In the developed dynamic procedure, central difference integration is used to integrate the equations of motion explicitly through time, using the kinematic conditions at the current increment  $i$  to calculate the kinematic conditions at the next increment  $i+1$ . That is,

$$\begin{aligned}\dot{U}_{(i+\frac{1}{2})}^N &= \dot{U}_{(i-\frac{1}{2})}^N + \frac{\Delta t_{i+1} + \Delta t_i}{2} \ddot{U}_{(i)}^N \\ U_{(i+1)}^N &= U_{(i)}^N + \Delta t_{(i+1)} \dot{U}_{(i+\frac{1}{2})}^N\end{aligned}\quad (1)$$

Where  $U^N$  and  $\dot{U}^N$  are the displacement and velocity degrees of freedom,  $\Delta t$  is the time step and the subscript  $i$  refers to the current increment number of dynamic steps. The key to the computational efficiency of the explicit procedure is the use of diagonal elements of mass matrices because the acceleration at the beginning of the increment is computed by:

$$\ddot{U}_{(i)}^N = (M^N)^{-1} (P_{(i)}^J - I_{(i)}^J - D_{(i)}^J) \quad (2)$$

Where,  $M$  is the mass matrix,  $P$  is the applied load vector,  $I$  and  $D$  are the internal and damping force vectors, respectively. In this procedure the time increments must be quite small, so that the accelerations are nearly constant during an increment. Fortunately, each increment is computationally inexpensive because there are no simultaneous equations needing to be solved.

Taking the nonlinearity and damping into account, the selection of time increments for the explicit central difference method is governed by the stability limit:

$$\begin{aligned}\Delta t &= \mu \text{Min}_{n=1}^{NE} \left( \frac{L}{C_p} \right) (\sqrt{\xi^2 + 1} - \xi); \\ C_p &= \text{Min}_{n=1}^{Nseg} \left( \sqrt{\frac{E_n(1-\nu)}{(1+\nu)(1-2\nu)\rho_n}} \right)\end{aligned}\quad (3)$$

Where  $\xi$  is the damping ratio and  $\mu$  is the reduction factor for a nonlinear system (normally,  $\mu = 0.2-0.6$  based on experience),  $E_n$  is the Young's modulus,  $\rho_n$  is the density,  $\nu$  is the Poisson's ratio of a segment. In order to determine an unstable solution, an energy balance check is also incorporated in this procedure. An unstable solution can easily be detected by the energy balance check, as any instability would result in the spurious generation of energy, which leads to a violation of the principle of conservation of energy.

### 3. SWITCH BETWEEN STATIC AND DYNAMIC PROCEDURES

The static and dynamic analyses are used as alternatives, to cover stable and unstable states. The static analysis can have advantages when the temperature is evolving and the structure is stable. As soon as the stability of the structures is lost, the dynamic procedure can be switched on automatically and the analysis can be carried on. If the stability of the structure is regained, the static analysis is triggered once again. The analysis will be kept going until the final failure of structure.

### Box 1. Flowchart for explicit time integration

1. Initial Conditions and initialization:

Set initial value of material state variables and  $U_0, \dot{U}_0$ , compute mass matrix  $M$  and initially estimate the time step.

2. Initialise the nodal internal force.
3. Compute the accelerations  $\ddot{U}_{(i)}^N = (M^N)^{-1}(P_{(i)}^J - I_{(i)}^J - D_{(i)}^J)$
4. Time update:  $t^{i+1} = t^i + \Delta t^{i+1/2}$ ,  $t^{i+1/2} = \frac{1}{2}(t^i + t^{i+1})$
5. First partial update nodal velocities:  $\dot{U}_{(i+1/2)}^N = \dot{U}_{(i)}^N + (t^{i+1/2} - t^i)\ddot{U}_{(i)}^N$
6. Enforce boundary conditions.
7. Update the nodal displacements:  $U_{(i+1)}^N = U_{(i)}^N + \Delta t^{i+1/2}\dot{U}_{(i+1/2)}^N$
8. Calculate the nodal internal forces.
9. Compute  $\ddot{U}_{(i+1)}^N$
10. Second partial update nodal velocities:  $\dot{U}_{(i+1)}^N = \dot{U}_{(i+1/2)}^N + (t^{i+1} - t^{i+1/2})\ddot{U}_{(i+1)}^N$
11. Check energy balance at time step  $i+1$
12. Adaptive check for variable time step.
13. Update counter:  $i=i+1$
14. Output; if simulation not complete, go to 4.

## 4. VALIDATION

### 4.1 Validation for dynamic procedure

The Williams toggle frame, shown in Fig. 1, has the structural behaviour known as snap-through buckling. The structure is assumed to be elastic and the material properties indicated in the figure have been used. Fixed bases and a rigid connection at the apex are assumed. The concentrated load at the apex of the frame increases linearly. Fig.1 shows a comparison of the results obtained by the current analysis with the numerical results of Yang and Chiou [9], together with the test results by Williams [10].

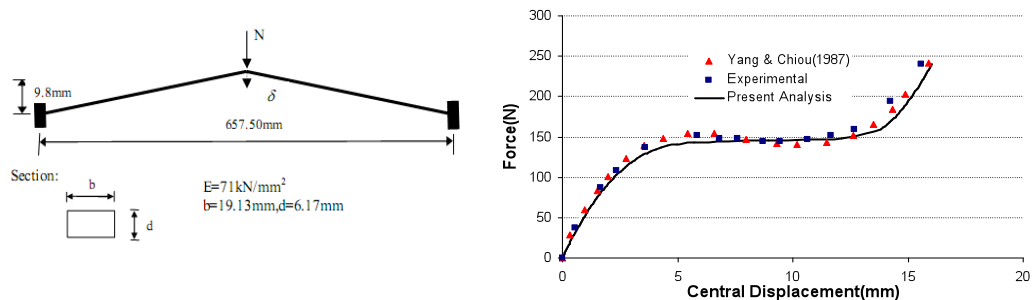


Fig.1 Comparison of predicted results together with test data

## 4.2 Validation of static/dynamic procedure

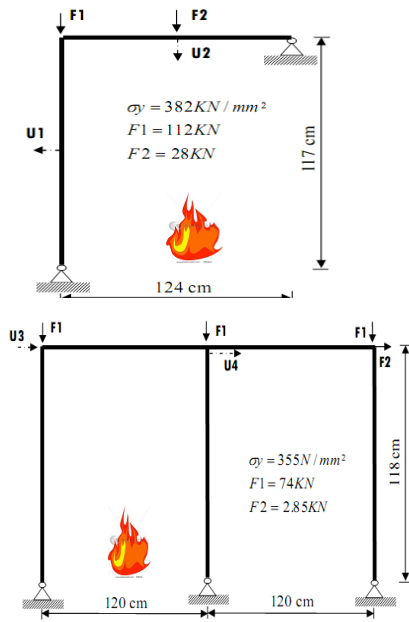


Fig.2 Configuration of EHR

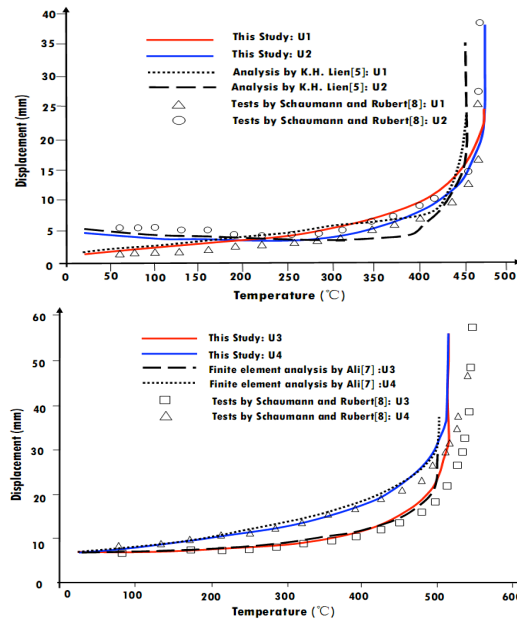


Fig.3 Comparison of predicted results with test data

The model has been validated using data from two groups of steel frames, tested by Rubert and Schaumann [7] and subsequently investigated by other researchers. Structural systems, dimensions and material properties of the frames are shown in Fig.2. The frames are all uniformly heated. The L-shaped frame is designated as EHR, and the double-span one is designated as ZSR. All structural elements were made of IPE80 I-sections. A comparison of the test and numerical results is shown in Fig.3.

To test the switch between the static and dynamic procedures, the simple frame shown in Fig.4 has been investigated under the effect of an ISO843 fire. The frame was also analyzed with the commercial FEA software ABAQUS/Standard to validate the developed model. Implicit integration was used in the ABAQUS model and the static/dynamic procedure was used in the *Vulcan* model. As indicated in Fig. 4, results from current model agree well with those from ABAQUS.

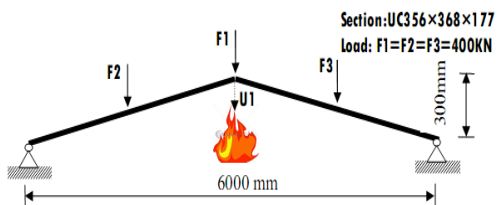
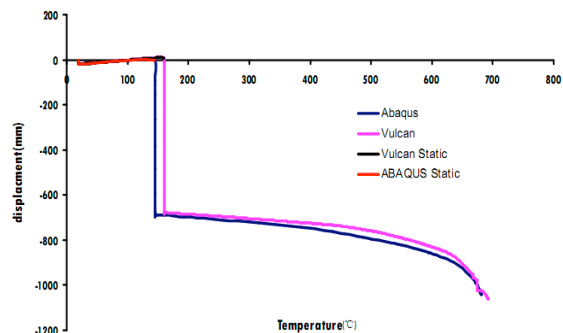


Fig.4 The latticed frame and comparison of predicted results.



## 5. ILLUSTRATION

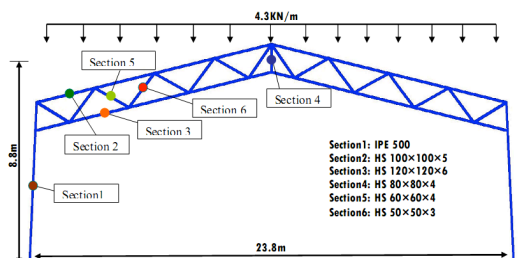


Fig.5 Configuration and loading conditions of frame.

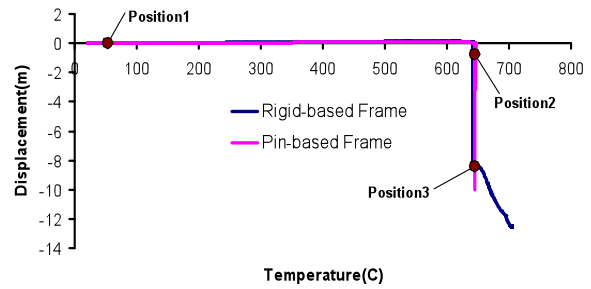


Fig.6 Apex displacement at elevated

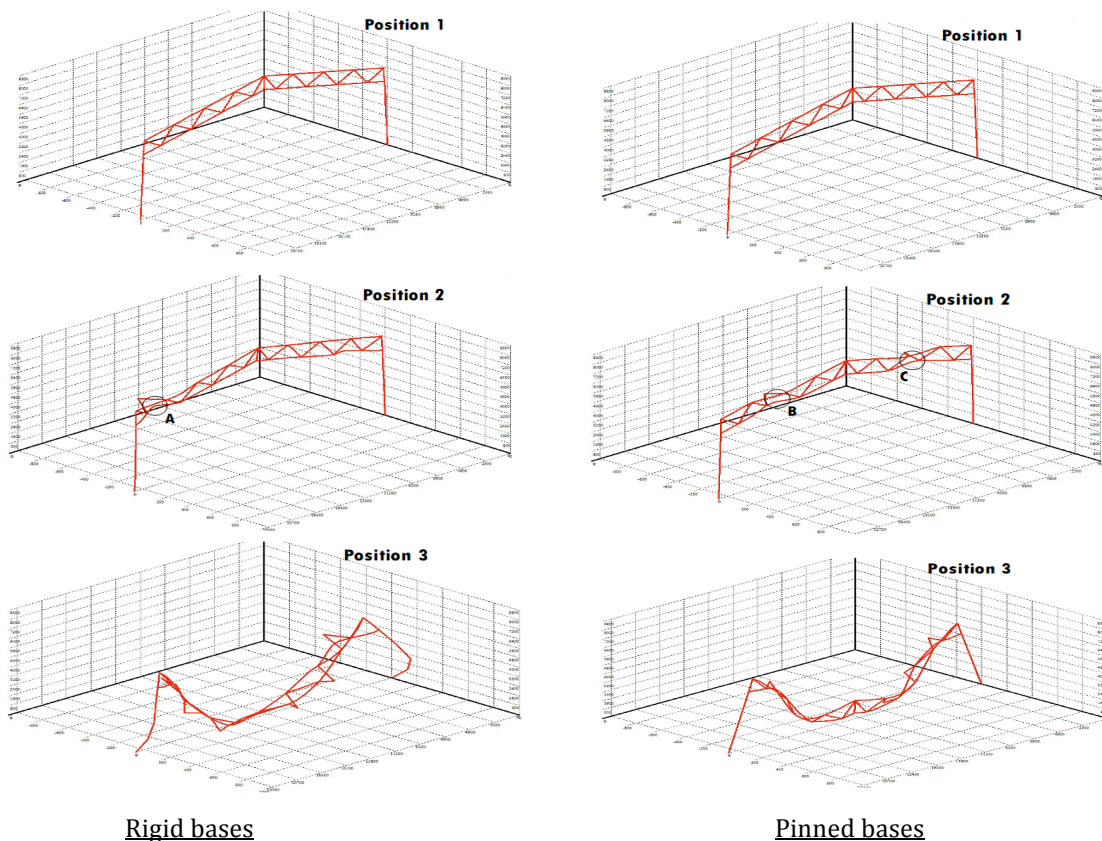


Fig.7 Deformation profiles

A uniformly heated latticed frame has been studied with the developed procedure. The dimensions and material properties shown in Fig. 5 are adopted and two different cases with rigid and pinned bases were tested. The structure experienced extremely large bending during the deformation; the apex deformation/temperature plot is shown in Fig. 6.

It can be seen that the rigid-based frame re-stabilized, in contrast to the pinned-based one. The collapse mechanisms differ as the boundary conditions change, even though the critical collapse temperature does not change much. For the rigid-based frame the failure is initiated by member buckling at location A as shown in Fig. 7; however, the failure begins at locations B and C in the pin-based frame.

## 6. CONCLUSION

A new full dynamic procedure has been developed for *Vulcan*, and has also been combined with the previous static version to extend the capability of *Vulcan* to trace behaviour from the local to global failure of a structure under fire conditions. As validated above, both the dynamic and static/dynamic analysis can properly predict the structural phenomena involved. The capability to trace large deformation has also been shown by the illustrative example. It is believed that this procedure is essential and will be effective for progressive collapse analysis under fire conditions. The slab element and simple connection elements have been incorporated into this procedure. Thus, the development enhances the prospect of directly determining the robustness of steel/composite structures under fire in design.

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